**AMC 8 Preparation Class**
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# Problem Set 10: Number Theory

Complete at least 30♣ in this homework set. For complete mastery get 40♣ or more.

**Problem 10.1** (2♣) Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82, and 91. What was the last score Mrs. Walter entered?

**Problem 10.2** (3♣) The digits 1, 2, 3, 4, 5, 6, 7, and 9 are used to form four two-digit prime numbers, with each digit used exactly once. What is the sum of these four primes?

**Problem 10.3** (2♣) Suppose that $m$ and $n$ are positive integers such that $75m = n^3$. What is the minimum possible value of $m + n$?

**Problem 10.4** (4♣) Marcy buys paint jars in containers of 2 and 7. What's the largest number of paint jars that Marcy can't obtain?

**Problem 10.5** (3♣) Find the remainder when the difference between 60002 and 601 is divided by 6.

**Problem 10.6** (4♣) How many positive two-digits integers are factors of $2^{24} - 1$ ?

**Problem 10.7** (5♣) Let $w$, $x$, $y$, and $z$ be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?

**Problem 10.8** (5♣) For how many integers $n$ between 1 and 1990, is the improper fraction $\frac{n^2+7}{n+4}$ not in lowest terms?

**Problem 10.9** (6♣) Let $x$ and $y$ be two-digit integers such that $y$ is obtained by reversing the digits of $x$. The integers $x$ and $y$ satisfy $x^2 - y^2 = m^2$ for some positive integer $m$. What is $x + y + m$?

**Problem 10.10** (6♣) A high school basketball game between the Raiders and the Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

**Problem 10.11** (8♣) Among the first 101 positive perfect squares, one is chosen at random. What is the probability that the chosen number leaves a remainder of 1 when divided by 12?

**Problem 10.12** (8♣) How many ordered pairs $(m, n)$ of positive integers, with $m \geq n$ have the property that their squares differ by 96?

**Problem 10.13** (10♣) For how many positive integers $n$ less than or equal to 24 is $n!$ evenly divisible by $1 + 2 + \ldots + n$?

# Questions

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